Q.P. Code – 22437

Fourth Semester B.Sc. Degree Examination, April/May 2019

(CBCS Scheme)

Paper IV – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answers ALL the questions.

SECTION – A

I. Answer any **FIFTEEN** of the following. **(15 × 2 = 30)**

1. Define normal subgroup.

2. If H is a normal subgroup of G, then prove that the product of any two right cosets of H is again a right coset.

3. Define homomorphism of groups.

4. Prove that an isomorphic image of an abelian group is also abelian.

5. Find the inverse of $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$.

6. Find the critical points of $f(x, y) = \sin(x + y)$.

7. By Maclaurian's expansion show that $e^x \cdot \log(1 + y) = y + xy + \cdots$

8. State Taylor's theorem for the function of two variables.

9. Define Gamma function.

10. Show that $\Gamma(n + 1) = n!$.

11. Evaluate $\int_0^{\infty} x^2 e^{-x} dx$, using gamma function.

12. Solve, $(D^2 + g)y = 0$.

13. Find the particular integral of $(D^2 + 4)y = \sin 2x$.

14. Find the part of complimentary function of $xy'' - (2x + 1)y' + (x + 1)y = x^2 e^x$.

15. Show that $(y + z)dx + (z + x)dy + (x + y)dz = 0$ is integrable.

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16. Evaluate $L\{\cos^2 t\}$.

17. Evaluate $L^{-1}\left\{\frac{S}{(S+3)^2}\right\}$

18. Evaluate $L^{-1}\left\{\frac{S+2}{S^2+4S+13}\right\}$.

19. Prove that intersection of two convex set is a convex set.

20. Define feasible solution and basic solution of LPP.

II. Answer any **TWO** of the following : **(2 × 5 = 10)**

21. Prove that H is a normal subgroup of G if and only if $gHg^{-1} = H$.

22. Show that the set $\frac{G}{H} = \{Ha / a \in G\}$ is a group w.r.t binary operation $H_a \cdot H_b = H_{ab}$, $\forall\ H_a,\ H_b \in \frac{G}{H}$, where H is a normal subgroup of G.

23. If $f : G \to G'$ is a homomorphism then prove that (a) $f(e) = e'$ (b) $f(a^{-1}) = [f(a)]^{-1}\ \forall\ a \in G$.

24. If $f : G \to G'$ is a homomorphism then prove that f is one-one, if and only if $K = \{e\}$.

III. Answer any **THREE** of the following : **(3 × 5 = 15)**

25. Expand $e^x \sin y$ by Taylr's expansion upto 3rd degree terms at (0, 0).

26. Find the extremum value of $f(x \cdot y) = x^3 y^2 (12 - x - y)$.

27. Find three numbers X, Y, Z such that $x + y + z = 1$ and $xy + yz + zx$ is maximum.

28. Prove that $(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where $m > 0$, $n > 0$.

29. Evaluate $\int_0^{\pi/2} \sin^4\theta \cos^2\theta \cdot d\theta$ using beta function.

IV. Answer any **THREE** of the following : **(3 × 5 = 15)**

30. Solve, $(D^3 - 1)y = 3 + e^{-x} + 5e^{2x}$.

31. Solve $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = x^2$.

32. Solve $\dfrac{d^2y}{dx^2} + \tan x \cdot \dfrac{dy}{dx} + 4y\cot^2 x = 0$ by changing the independent variable.

33. Show that $(1 + x^2)y'' - 4xy' + 2y = \sec^2 x$ is exact and hence solve.

34. Solve $\dfrac{dx}{mz - ny} = \dfrac{dy}{nx - lz} = \dfrac{dz}{ly - mx}$.

V. Answer any **TWO** of the following : **(2 × 5 = 10)**

35. Evaluate :

(a) $L\{(t+1)^2 e^{3t}\}$

(b) $L\{\sin 5t \cdot \cos 2t\}$.

36. Find :

(a) $L\left\{\dfrac{\sin t}{t}\right\}$

(b) $L^{-1}\left\{\dfrac{1}{(s+1)(s+2)}\right\}$.

37. Solve $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} - 3y = \sin t$ where $y(0) = y'(0) = 0$ using Laplace transformers.

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VI. Answer any **TWO** of the following : **(2 × 5 = 10)**

38. Show that the set $S = \{(x_1, x_2) / 2x_1 + 3x_2 = 7\}$ is a convex in R^2

39. Solve graphically the following system of inequalities $2x + y \geq 3$, $x - 2y \leq -1$, $y < 3$.

40. Use Simplex method to solve the following LPP :

    Maximize $Z = x - y + 3z$

    Subject to the constraints

    $x + y + z \leq 10$
    $2x - z \leq 2$
    $2x - 2y + 3z \leq 0$
    $x, y, z \geq 0$