Q.P. Code – 42439

Fourth Semester B.Sc. Degree Examination, April/May 2019

(CBCS Scheme)

Mathematics

Paper 4.1 – ALGEBRA AND CALCULUS – 2

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answers ALL the questions.

I. Answer any **SIX** of the following. **(6 × 2 = 12)**

1. Evaluate $\int_c (x+y)dx + (y-x)dy$ along the parabola $y^2 = x$ from (1, 1) to (4, 2).

2. Evaluate $\int_0^3 \int_1^2 (x^2 + 3y^2)dy\, dx$.

3. If $r = \sqrt{x^2 + y^2 + z^2}$ find ∇r.

4. Prove that $\text{div}(\text{grad } \phi) = \nabla^2 \phi$.

5. Show that the subset $H = \{1, -1\}$ is a subgroup of group $G = \{1, -1, i, -i\}$ with respect to multiplication.

6. Find the number of generators of a cyclic group of order 60.

7. Prove that every subgroup of an Abelian group is normal subgroup.

8. Define endomorphism and automorphism.

II. Answer any **SIX** of the following : **(6 × 3 = 18)**

9. Evaluate $\int_0^1 \int_0^{x^2} e^{y/x} dy\, dx$.

10. Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2 yz\, dz\, dy\, dx$.

11. Find the unit vector normal to the surface $x^2 + 2y - 3z = 5$ at (1, 2, 0).

12. Show that $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational.

13. If H is a subgroup of group G then prove that $H = H^{-1}$.

14. Prove that every cyclic group is an Abelian group.

15. Prove that intersection of two normal subgroups of a group G is normal subgroup of G.

16. If $f : G \to G'$ defined by $f(x) = \log_{10} x$, $\forall x \in G$ where G is a multiplicative group of positive real numbers and G' is the additive group of real numbers. Verify f is homomorphism. Also find Kernal at homomorphism.

III. Answer any **THREE** of the following. **(3 × 5 = 15)**

17. Evaluate $\int_C [(x+2y)dx + (4-2x)dy]$ around the ellipse whose equation is $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

18. Evaluate $\iint_S x^2 dx\, dy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.

19. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\, dx$ by changing the order of integration.

20. Evaluate $\iiint_V xyz\, dz\, dy\, dx$ where v is bounded by the plane $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

IV. Answer any **THREE** of the following : **(3 × 5 = 15)**

21. Find the angle of intersection of the sphere $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z - 47 = 0$ at (4, -3, 1) common to them.

22. Prove that $div(r^3 \cdot \vec{r}) = 6r^3$ where $\vec{r} = \hat{x}_1 + \hat{y}_1 + z\hat{k}$.

23. If ϕ is a scalar function and $\vec{A}$ is a vector function then prove that

$$\text{curl}\,(\phi \vec{A}) = \phi(\text{curl}\, \vec{A}) + [\text{grad}\, \phi] \times \vec{A}$$

24. State and prove Green's theorem.

V. Answer any **THREE** of the following : **(3 × 5 = 15)**

25. Prove that $G = \{1, 5, 7, 11\}$ is an abelian group with respect to multiplicative modulo 12.

26. If H and K are any two subgroups of a group G then prove that HK is a subgroup iff $HK = KH$.

27. In a group G, $\forall\ a \in G$ then prove that $0(a) = 0(a^{-1})$.

28. State and prove Lagrange's theorem.

VI. Answer any **THREE** of the following : **(3 × 5 = 15)**

29. Prove that A subgroup H of a group G is normal iff every right coset of H in G is a left coset of H in G.

30. If N is a normal subgroup of G and H is only subgroup of G, then prove that NH is a subgroup of G.

31. If $f : G \to G'$ is homomorphism of groups then prove that

    (a) $f(e) = e'$ where $e \in G$ and $e' \in G'$

    (b) $f(a^{-1}) = [f(a)]^{-1} \forall\ a \in G$

32. If $f : G \to G'$ be a homomorphism of groups with Kernal K then prove that K is normal subgroup of G.
