Sixth Semester B.Sc. Degree Examination, April/May 2019

(CBCS Scheme)

Mathematics

Paper 6.1 – COMPLEX ANALYSIS AND NUMERICAL METHODS

Time : 3 Hours]   *[Max. Marks : 90*

Instructions to Candidates : Answers ALL the questions.

PART – A

I.   Answer any **SIX** of the following.   **(6 × 2 = 12)**

1.   Find the real and imaginary part of $e^{\frac{i\pi}{2}}$.

2.   Show that $u = e^x \sin y$ is a harmonic function.

3.   Define power series.

4.   Evaluate $\int_{(0,1)}^{(2,5)} ((3x+y)dx + (2y-x)dy)$ along the curve $y = x^2 + 1$.

5.   Evaluate $\int_C \frac{e^z}{(z-1)}\,dz$ where $c : |z| = 2$.

6.   Prove that $\Delta = E - 1$.

7.   Evaluate $\Delta^3\left[(1+2x)(1+4x)(1+6x)\right]$ by taking $h = 1$.

PART – B

II.   Answer any **SIX** of the following :   **(6 × 3 = 18)**

8.   Show that $|z|^2 = 4\,\text{Re}(z+2)$ represents a circle. Find its centre and radius.

9.   Evaluate $\bigcup\limits_{z \to e^{\frac{i\pi}{4}}} \left(\frac{z^2}{z^4 + z^2 + 1}\right)$.

10.   Construct the analytic function where real part is $e^x \sin y$.

11. Evaluate $\int\limits_{C} \dfrac{e^{2z}}{(z-2)^2} dz$ where $C : |z| = 3$.

12. Evaluate $\int\limits_{C} \left[(x+2y)dx + (4-2x)dy \right]$ around the ellipse $x = 4\cos\theta$, $y = 3\sin\theta$ where $0 \le \theta \le 2\pi$.

13. Construct a difference table for $f(x) = x^2 + x + 1$ for the values $x = 0(1)4$.

14. Using Simpson's 1/3 rule evaluate $\int\limits_{1}^{5} f(x)\, dx$. Given that

x	1	2	3	4	5
$f(x)$	13	50	70	80	100

PART C

III. Answer any **FOUR** of the following : **(4 × 5 = 20)**

15. State and prove necessary condition for the function $f(z)$ to be analytic.

16. Show that $\arg\left(\dfrac{z-1}{z+2} \right) = \dfrac{\pi}{3}$ represents a circle. Find the centre and radius.

17. Show that $f(z) = \sin z$ is analytic and also find its derivative.

18. Find the orthogonal trajectories of family of curves $x^2 - y^2 + x = G$.

19. If $f(z) = u + iv$ is analytic where u and v are harmonic function then prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} \right) \left| \operatorname{Re} f(z) \right|^2 = 2\left| f'(z) \right|^2$.

IV. Answer any **FOUR** of the following : **(4 × 5 = 20)**

20. Evaluate $\int\limits_{0}^{1+i} (x^2 - iz)dz$ along $y = x$ and $y = x^2$.

21. State and prove Cauchy's integral theorem.

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22. Evaluate $\displaystyle\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)}\, dz$ where $c : |z| = 3$.

23. Evaluate $\displaystyle\int_C \frac{\sin^2 z}{\left(z - \dfrac{\pi}{6}\right)^3}\, dz$ where $c : |z| = 1$.

24. State and prove Liouville's theorem.

V. Answer any **FOUR** of the following : $\hspace{2cm}$ (4 × 5 = 20)

25. Use the method of separation of symbols to prove

$$u_0 - u_1 + u_2 - u_3 + \cdots = \frac{1}{2} u_0 - \frac{1}{4} \Delta u_0 + \frac{1}{8} \Delta^2 u_0 - \frac{1}{16} \Delta^3 u_0 + \cdots$$

26. From the following table find the number of students who obtained < 45 marks.

Marks	30-40	40-50	50-60	60-70	70-80
Number of Students	31	42	51	35	31

27. Normal weight of baby during the 1st eight months of life.

Age in Months	0	2	5	8
Weight in months :	6	10	12	16

Estimate the weight of the baby at the age of 7 months using Lagrange's interpolation formula.

28. Evaluate $\displaystyle\int_{-\pi/2}^{\pi/2} \cos x \, dx$ taking '6' subintervals by Weddle's rule.

29. Find $f'(6)$ and $f''(6)$ from the following table :

x	2	4	6	8	10
$f(x)$	4	12	19	52	84

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