Q.P. Code – 42634

Sixth Semester B.Sc. Degree Examination, April/May 2019

(CBCS Scheme)

Mathematics

Paper VIII – 6.2(a) – NUMBER THEORY

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answers ALL the questions.

SECTION – A

I. Answer any **SIX** of the following. **(6 × 2 = 12)**

1. If a/b and b/c then prove that a/c where a, b, c are integers.

2. Prove that every odd integer is of the form $4q \pm 1$.

3. Write the fundamental theorem arithmetic.

4. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then prove that $ac \equiv b\,d \pmod{m}$.

5. Find the digit in the unit place of the number 3^{101}.

6. Define multiplicative function. Give an example.

7. Find the number of positive integers ≤ 3600, that are co-prime to 3600.

SECTION – B

II. Answer any **SIX** of the following : **(6 × 3 = 18)**

8. Prove that the number of primes is infinite.

9. If 6/36, write the reason for failure of Euclid's lemma.

10. Find the solution of Diophatine equation $70x + 112y = 168$.

11. Find the remainder when 2^{100} is divided by 13.

12. Define Fermat's number, write the first four Fermat's number.

13. Find the positive divisor of 1026.

14. For each integer $\eta \geq 1$, prove that $\sum_{d/n} \mu(d) = \begin{cases} 1 \text{ if } \eta = 1 \\ 0 \text{ if } \eta > 1 \end{cases}$.

SECTION – C

III. Answer any **FOUR** of the following. **(4 × 5 = 20)**

15. Prove that the product of any three consecutive integers is divisible by 3 !

16. Find the G.C.D. of 527 and 765. And express in the form $527x + 765y$.

17. If $4x - y$ is a multiple of 3, then show that $4x^2 + 7xy - 2y^2$ is divisible by 9.

18. If a and b are any two integers not both zero then show that GCD of a and b exists and its unique.

19. If (x_0, y_0) is one solution of $ax + by = c$ and $(a,b) = d$, then prove that the general solution $x_1 = x_0 - \frac{b}{d}t$, $y_1 = y_0 + \frac{a}{d}t$, $(t \in z)$.

IV. Answer any **FOUR** of the following : **(4 × 5 = 20)**

20. If $a \equiv b(\bmod m)$ and $c \equiv d(\bmod m)$ then prove that

(a) $a + c \equiv b + d(\bmod m)$

(b) $a^n \equiv b^n(\bmod m)$

21. Find the remainder when the sum $S = 1! + 2! + 3! + \cdots + 1000\,!$ is divided by 8.

22. Solve the linear congruence $6x \equiv 15(\bmod 2)$.

23. State and prove Fermat's Little's theorem.

24. Show that 17 is prime by showing that $16! \equiv -1(\bmod 17)$.

V. Answer any **FOUR** of the following : **(4 × 5 = 20)**

25. If $\eta = P_1^{K_1}, P_2^{K_2} \cdots P_r^{k_r}$ is the prime factorization $(n>1)$ then prove that

(a) $\tau(n) = (K_1+1)(K_2+1)\cdots(K_r+1)$

(b) $\sigma(n) = \left(\dfrac{p_1^{k_1+1}-1}{p_1-1}\right)\left(\dfrac{p_2^{k_2+1}-1}{p_2-1}\right)\cdots\cdots\left(\dfrac{p_r^{k_r+1}-1}{p_r-1}\right)$.

26. Show that $\phi(\eta) = \phi(\eta+1) = \phi(\eta+2)$ for $\eta = 5186$.

27. If $\eta = P_1^{K_1}, P_2^{K_2} \cdots P_r^{k_r}$ then prove that $\sum \dfrac{\mu(d)}{\eta} = \left(1-\dfrac{1}{p_1}\right)\left(1-\dfrac{1}{p_2}\right)\cdots\left(1-\dfrac{1}{p_r}\right)$.

28. Find the number of positive divisors and sum of all positive divisors of 39744.

29. If p and $2p+1$ are both prime and $n=4p$ then show that $\phi(n+2)=\phi(n)+2$.