Q.P. Code – 22337

Third Semester B.Sc. Degree Examination, October/November 2019

(Semester Scheme)

Paper III – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answers ALL questions. Answers should be written completely in English.

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Define the subgroup of a group.

2. Find the order of each element of the multiplicative group $G=\{1,-1,i,-i\}$.

3. Show that the group $G=\{1,w,w^2\}$ of the cube roots of unity w.r.t. multiplication is a cyclic group.

4. Show that '1' is a generator of the group $(z_n,+_n)$.

5. Define the index of a subgroup *H* of a group *G*.

6. Find all the subgroups of the group $(Z_{10},+_{10})$.

7. Define infimum and supremum of a sequence.

8. Find the limit of the sequence $\left\{n\sin\left(\frac{1}{n}\right)\right\}$.

9. Test the convergence of the sequence $\{1-(-1)^n\}$.

10. Show that the series $\sum_{n=1}^{\infty}\frac{1}{(n+1)^2}$ is convergent.

11. State Cauchy's root test for convergence of a series of positive terms.

12. Test the nature of the series $\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n)}$.

13. Discuss the nature of the series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots$.

14. Find the sum to infinity of the series $1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\pm\cdots$.

15. If $g(x)=\begin{cases} x-2 & \text{for } x<0 \\ 2x-1 & \text{for } x>0 \end{cases}$, find $\lim_{x\to 0}[g(x)]$, if it exists.

16. State Lagrange's mean value theorem.

17. Show that $\cosh x = 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots$ using Maclaurin's series.

18. Using L'Hospital's rule, evaluate $\lim_{x\to 0}[x\log(\tan x)]$.

19. Write the Fourier expansion of an even function $f(x)$ defined in the interval $(-l,l)$.

20. Obtain the half range sine series expansion of $f(x)=x^2$ in (0, 2)

II. Answer any **THREE** of the following. **(3 × 5 = 15)**

21. Prove that a subset H of a group G is a subgroup of G if and only if $HH^{-1}=H$.

22. If 'a' is any element of a group of G, of order 'n', then prove that $a^m=e$ for any integer 'm' if and only if n divides m.

23. Define a cyclic group. Prove that every cyclic group is Abelian.

24. Prove that there exists a one-to-one correspondence between two right cosets of a subgroup H of a group G.

25. Prove that, if 'a' is any integer and 'p' is any positive prime, then $a^p \equiv a(\bmod p)$.

III. Answer any **TWO** of the following. **(2 × 5 = 10)**

26. If $\{a_n\}$ and $\{b_n\}$ are sequences such that $\lim_{n\to\infty} a_n = l$ and $\lim_{n\to\infty} b_n = m$, then prove that $\lim_{n\to\infty}(a_n \pm b_n) = l \pm m$.

27. Discuss the nature of the sequence $\{x^n\}$, $x \in \Re$.

28. Show that the sequence $\{x_n\}$, where $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots \frac{1}{n+n}$, is convergent.

IV. Answer any **THREE** of the following. **(3 × 5 = 15)**

29. Prove that, if a series $\sum_{n=1}^{\infty} u_n$ is convergent, then $\lim_{n\to\infty} u_n = 0$. Further, give an example to prove that the converse is not true.

30. State and prove D'Alembert's ratio test for convergence of a series of positive terms.

31. Test the nature of the series $\sum_{n=1}^{\infty}\left(\frac{nx}{n+1}\right)^n$.

32. Discuss the convergence of $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n}{2n-1}$.

33. Sum of infinity the series $\frac{3\cdot5}{3\cdot6} + \frac{3\cdot5\cdot7}{3\cdot6\cdot9} + \frac{3\cdot5\cdot7\cdot9}{3\cdot6\cdot9\cdot12} + \cdots$

V. Answer any **TWO** of the following : **(2 × 5 = 10)**

34. Show that $f(x) = \begin{cases} x^2 - 1 & \text{if } x > 1 \\ 1 - x & \text{if } x < 1 \end{cases}$ is not differentiable at $x = 1$.

35. State and prove Rolle's theorem.

36. Evaluate $\lim_{x\to0}\left[\frac{1}{x^2} - \cot^2 x\right]$ using L'Hospital's rule.

37. Evaluate $\lim_{x\to0}\left(\frac{a^x + b^x}{2}\right)^{1/x}$.

VI. Answer any **TWO** of the following : **(2 × 5 = 10)**

38. Obtain the Fourier series expansion of $f(x)=\begin{cases}-k & \text{if} \quad -\pi \le x < 0 \\ k & \text{if} \quad 0 \le x \le \pi\end{cases}$.

39. Expand $f(x)=x-x^2$ as a Fourier series in (-1, 1).

40. Obtain the half range cosine series expansion of $f(x)=\begin{cases}x & \text{if} \quad 0 \le x < \pi \\ 2\pi - x & \text{if} \quad \pi < x \le 2\pi\end{cases}$.
