Q.P. Code – 42339

Third Semester B.Sc. Degree Examination, October/November 2019

(CBCS Semester Scheme)

Mathematics

Paper 3.1 – REAL ANALYSIS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answers ALL questions. Answer should be written completely in English.

PART – A

Answer any **SIX** questions : **(6 × 2 = 12)**

1. State rational density theorem.

2. If $ab = 0$ then prove that either $a = 0$ or $b = 0$.

3. Define convergence and oscillatory sequence.

4. If $\{a_n\}$ is convergent sequence of positive term then evaluate $\lim_{n\to\infty} a_n$ where $a_{n+1} = \frac{6}{5 + a_n}$.

5. Write the condition for the convergence and divergence of the geometric series $\sum_{n=0}^{\infty} x^n$.

6. Test the convergence of the series $\sum \left(\frac{n}{n+1}\right)^{n^2}$.

7. State Rolle's theorem for the function $f(x)$.

8. Evaluate $\lim_{x\to 0} \frac{x - \sin x}{x^3}$.

PART – B

II. Answer any **SIX** questions : **(6 × 3 = 18)**

9. Prove that intersection of two neighbourhoods of a point is also a neighbourhood of that point.

10. Prove that intersection of two open set is an open set.

11. Find the nature of the sequence $a_n = \left(\frac{n-3}{n+2}\right)^{n/3}$.

12. Verify whether the sequence $\{a_n\}$ is monotonic increasing or decreasing where $a_n = \frac{n+3}{n+4}$.

13. Test the convergence of the series $\sum \frac{1}{n}\tan(1/n)$.

14. Discuss the convergence of the series $\sum (-1)^{n-1} \cdot \frac{n}{2n-1}$.

15. Verify Cauchy's mean value theorem for e^x and e^{-x} in $[a,b]$.

16. Expand $\log_e^x$ about $x = 1$ by Taylor's series.

PART – C

III. Answer any **THREE** questions : **(3 × 5 = 15)**

17. Prove that every subset of countable set is countable.

18. Prove that every non empty subset of real numbers is bounded above has the least upper bound.

19. State and prove Archimedean property of real numbers.

20. Let A be a closed set and B be an open set, show that

(a) $B - A$ is an open set

(b) $A - B$ is a closed set.

PART – D

IV. Answer any **THREE** questions : **(3 × 5 = 15)**

21. Discuss the behaviour of the sequence whose n th term are

(a) $n[\log(n+1)-\log n]$

(b) $\frac{(n+1)^{n+1}}{n^n}$.

22. Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}$ is convergent.

23. Prove that every monotonically increasing sequence and bounded above is convergent.

24. Discuss the nature of the sequence $\{x^n\}$ where x is a real number.

PART – E

V. Answer any **THREE** questions : **(3 × 5 = 15)**

25. Let $\sum u_n$ and $\sum v_n$ be two series of positive terms and $\lim_{n\to\infty}\frac{u_n}{v_n}$ be a finite non zero quantity then prove that $\sum u_n$ and $\sum v_n$ both converge or diverge.

26. Discuss the convergence of the series $\sum \frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot4\cdot6\cdots2n}x^n$.

27. State and prove Raabe's test for positive series.

28. Discuss the convergence of the series $\frac{x}{1\cdot3}+\frac{x^2}{3\cdot5}+\frac{x^3}{5\cdot7}+\cdots$

PART – F

VI. Answer any **THREE** questions : **(3 × 5 = 15)**

29. If $f(x)$ is continuous in $[a,b]$ and $f(a) \neq f(b)$, then prove that $f(x)$ takes every value between $f(a)$ and $f(b)$ atleast once.

30. State and prove Lagrange's mean value theorem.

31. Expand $\log(1+x)$ upto term containing x^4 using Maclaurin's series.

32. Evaluate $\lim_{x \to 0} \left[\frac{a^x + b^x}{2} \right]^{\frac{1}{x}}$.