Q.P. Code – 42533

Fifth Semester B.Sc. Degree Examination, October/November 2019

(CBCS Scheme)

Mathematics

Paper 5.1 – ADVANCED ALGEBRA AND NUMERICAL METHODS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1. *Answers ALL the questions.*
2. *Answer should be written completely in English.*

PART – A

I. Answer any **SIX** of the following : **(6 × 2 = 12)**

1. Define skew field. Give an example for a skew field.

2. Show that $S = \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \Big/ a, b \in z \right\}$ is a subring of $M_2(Z)$.

3. If I be a ideal of a ring R with unity $1 \in I$ then prove that $I = R$.

4. Define vector space over a field F.

5. Prove that the set of vectors of $V(F)$ containing the zero vector is linearly dependent.

6. Solve the equation $x^3 - 4x - 9 = 0$ in (2, 3) by bisection method in two steps.

7. Explain Jacobi-iteration method to solve the system of three equations.

PART – B

II. Answer any **SIX** of the following : **(6 × 3 = 18)**

8. Let $(R, +, \cdot)$ be a ring $\forall\, a \in R$, then prove that $a \cdot 0 = 0 \cdot a = 0$ where 0 is the additive identity.

9. Show that $S = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \Big/ a, b \in Z \right\}$ is neither left ideal nor right ideal.

10. Prove that Kernel of homomorphism of ring is a subring.

11. Show that $W = \{(x,0,0) / x \in R\}$ is a subspace of $V_3(R)$.

12. If $T : V_3(R) \to V_3(R)$ defined by $T(x, y, z) = (0, y, z)$ then show that T is a linear transformation.

13. Find the cube root of 54 correct to three places of decimal by Newton-Raphson method.

14. Solve the equations $10x + y + z = 12$, $2x + 10y + z = 13$, $2x + 2y + 10z = 14$ by Gauss-Seidal iteration method up to three iterations.

PART – C

III. Answer any **FOUR** of the following : **(4 × 5 = 20)**

15. Prove that the ring of integers module 'n' $(z_n, +_n, X_n)$ is an integral domain if and only if 'n' is a prime number.

16. Let $(R, +, \cdot)$ be a ring, a non empty set S of a ring R is a subring of R then prove that

    (a) $\forall\, a, b \in S \Rightarrow a - b \in s$

    (b) $\forall\, a, b \in S \Rightarrow a \cdot b \in s$.

17. If $f : R \to R'$ be a homomorphism with Kernel k then prove that f is one-one iff $K = \{0\}$.

18. If I be an ideal of a ring R then prove that

    (a) R is a commutative then R/I is also commutative

    (b) If R has unity then R/I is also unity.

19. State and prove fundamental theorem of homomorphism of rings.

PART – D

IV. Answer any **FOUR** of the following : **(4 × 5 = 20)**

20. Prove that the intersection of any two subspaces of a vector space is also a subspace but the union of two subspaces need not be subspace.

21. Express the vectors $(2, -5, -1)$ as a linear combination of the vectors $(1, 2, 3)$, $(2, 1, 1)$ and $(1, 3, 2)$ of $V_3(R)$.

22. Prove that in an n-dimensional vector space $V(F)$

    (a) any $(n+1)$ vectors of V are linearly dependent

    (b) no set of $(n-1)$ elements can span V.

23. Find the linear transformation $T: R^2 \to R^3$ such that $T(-1, 1) = (-1, 0, 2)$ and $T(2, 1) = (1, 2, 1)$.

24. Find the range space, null space, rank and nullity of a linear transformation $T: V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (y - x,\ y - z)$.

PART – E

V. Answer any **FOUR** of the following : **(4 × 5 = 20)**

25. Solve the equation $x^3 - x^2 - x - 3 = 0$ over (2, 2.5) by Newton-Raphson method correct to three places of decimal.

26. Solve the equation $x^4 - x - 10 = 0$ has one root between 1.8 and 2 correct to three places of decimal by Regula-Falsi method.

27. Solve the following equations by Gauss elimination method.

    $2x_1 + 4x_2 + x_3 = 3$, $3x_1 + 2x_2 - 2x_3 = -2$ and $x_1 - x_2 + x_3 = 6$.

28. Apply Euler's modified method to find y for $x = 0.05$ for the equation $\frac{dy}{dx} = x + y$ with $y(0) = 1$.

29. Apply Runge-Kutta method to solve the equation $\frac{dy}{dx} = 1 + \frac{y}{x}$ with $y(2) = 2$ for $x = 2.1$.